Teacher Tune-up

Quick Content Refresher for Busy Professionals

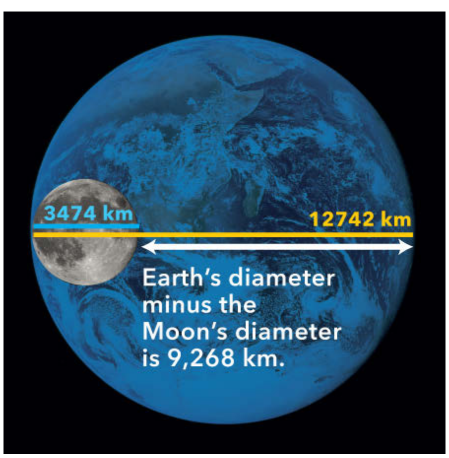
Ratios and Rates in Science

Ratios and rates—two ideas central to proportional thinking—appear throughout the middle grades in both mathematics and science. In mathematics, of course, they're closely tied to fractions. For a discussion of ratios and fractions from a mathematical standpoint, see math.serpmedia.org/poster_problems/ratio-v.-fraction.html. For a more science-specific treatment, read on!

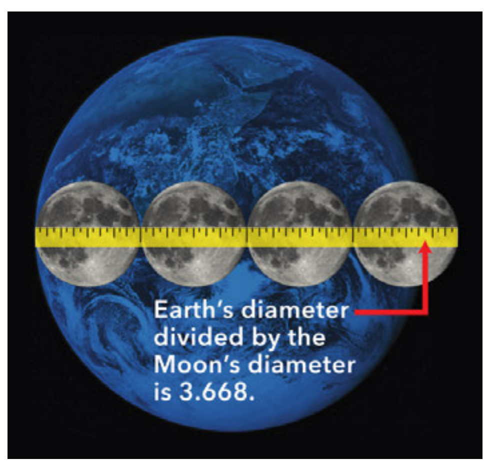
Ratios

We frequently make comparisons in science. For example, we might compare the size of the earth to the size of the moon. Here are the diameters.

The earth is bigger, but by how much? If we want to be quantitative about this comparison, and give it a number, we have two choices:



We can *subtract* (12742 km – 3474 km) to find out that the earth's diameter is 9,268 km *more than* the moon's diameter.



Or we can *divide* (12742 km ÷ 3474 km) and find out that the earth's diameter is 3.668 *times* the moon's diameter.

Both of these approaches are correct, but the second one is usually more useful. Often, ratios make things easier to picture: you get an accurate sense of how big the moon is by imagining 3 2/3 moons laid across the earth.

Measuring change works the same way. Suppose you have a plant that's 20 cm tall, and a week later, it's 24 cm tall. You could find the difference and say that it grew 4 cm; or you could find a ratio and say that it grew to 1.2 times its original size. We don't usually say it that way, though. We say it *grew by 20 percent*.

This last way—talking about a ratio in terms of a percentage—is both amazingly common and really confusing for students. Think of all the little calculations you do: 24 ÷ 20 = 1.2. But to isolate the part that represents growth, you subtract one, so you have 0.2. But that's an awkward number, so we convert it to percentage, multiplying by 100. And 100 times 0.2 is 20%.

One way to try to understand these computations is to combine the ideas of difference and ratio. That is, the difference is 4 cm. How big is that compared to the plant? The ratio in that case is 4 ÷ 20, which is 0.2. And that's 20%. This approach avoids the "subtract one" issue. In general, if you're computing percent change, you use the following kind of equation.

$$\text{percent change} = 100 \times \frac{(\text{new value} - \text{old value})}{\text{old value}}$$

In this formula, you can see the difference *and* the ratio. But wait a second: why use the ratio then? Why not just the difference?

You could. But here is what the ratio gets you: picture a tree 20 meters tall. That's 2000 cm. It grows by the same amount, 4 cm, to 2004 cm tall. Does that mean the tree grew as much as your plant? Yes and no. It's still a 4 cm gain—but while that 4 cm is insignificant for the tree, it's a big deal for the plant.

That is, the ratio puts that growth in perspective, literally in proportion to the plant.

We do exactly the same thing when we compare our estimates or results to a "true" value. Suppose Anabel picks up a potato and estimates that its mass is 222 grams. She weighs the potato on a classroom balance and gets 205 grams. Her "error" is the difference (17 grams) divided by the true value (205), or about 0.08. And in percent? Multiply by 100, or move the decimal point two to the left: Anabel guessed the potato's mass within about 8%.

$$\text{percent error} = 100 \times \frac{(\text{estimate} - \text{true value})}{\text{true value}}$$

One final note: the direction of the subtraction is important. If Anabel had guessed too low (say, 199 grams) the error would be negative (about –3%).

Rates

A rate is a special kind of ratio, and expresses a proportional relationship. The most common kind of rate is basically a speed. You can often express the units of a rate using the word "per."

Suppose Markko runs 100 meters in 16.6 seconds. If we make a ratio,

$$\frac{\text{100 meters}}{\text{16.6 seconds}}$$

We've made a rate. And if we actually perform the division (100 ÷ 16.6 is about 6), we get a *unit rate*,

$$6 \, \frac{\text{meters}}{\text{second}}$$

which we also write as 6 meters/sec, 6 m/s, 6 meters *per* second, or simply 6 mps.

That is, on the average, over those 100 meters, Markko covered 6 meters every second. Rates like this are useful because they let you compare. Suppose on another day, Markko ran 60 meters in 10.3 seconds. Sylvie thinks Markko ran faster because his run took less time. But Markko says, "Maybe not, because it was a shorter distance." Terry realizes they could make a ratio.

$$\frac{\text{60 meters}}{\text{10.3 seconds}} \approx \text{5.8 m/s}$$

Aha! Markko was slower, on average, on the shorter course that day.

How are these rates different from ratios? They're not really comparing two similar things like the ratio of sizes of the Earth and Moon (two giant balls) or the change in the height of a plant (the plant before and after). In fact, a typical rate is a ratio of two quantities *with different units*. Instead of looking at change or error or relative amount, a rate expresses a proportion: in the case of speed, how much distance is covered each second.

Let's see a few more examples.

- Kelly takes her pulse and gets 18 beats in 15 seconds. Because 18 ÷ 15 = 1.2, that's 1.2 *beats per second*. She can multiply that by 60 to get 72 *beats per minute*.
- Laurence brings 144 cookies to a class of 36 students. (What a guy!) 144 ÷ 36 = 4, so that's 4 *cookies per student*. Is this a rate? You bet. It's unusual, but it's okay that the "denominator" is not in units of time.
- Pat used 26 (small) oranges to make 1.3 liters of orange juice. 26 ÷ 1.3 = 20 *oranges per liter*.

You can then use rates to solve other problems. If Pat's class needs 6 liters of orange juice, they will need 120 oranges.

$$20\ \frac{\text{oranges}}{} \times 6 \text{ liters} = 120 \text{ oranges}$$

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Finally, you can use rates "flipped over." In Pat's case,

$$\frac{1.3 \text{ liters}}{26 \text{ oranges}} \approx 0.05 \text{ liters per orange}$$

Because 0.05 is a small, awkward number to use with a unit, we can convert from liters to milliliters. Then the rate is

$$0.05\ \frac{\text{liters}}{\text{orange}} \times \frac{1000 \text{ mL}}{\text{liter}} = 50 \text{ mL/orange}$$

which actually makes intuitive sense: a single small orange would yield a fairly small amount of liquid.

