Teacher Tune-up

Quick Content Refresher for Busy Professionals

Converting Units

Unit conversion can be a rats' nest for some students. It can be hard to remember which way it works. You find a conversion factor, but do you multiply or divide?

Students can be amazingly haphazard about this topic. Even high-school students will report that their feet are 28 km long. But to be fair, if they have little experience with a unit, it's no surprise that they can get ludicrous results and not notice. These students need systematic ways to convert units so they can develop their intuition about how big those units are.

In this tune-up, we'll discuss two reliable strategies for converting a quantity from one unit to another. Which one to choose may depend on the student. Both techniques work great as long as you're careful and skip steps only when you really know what you're doing.

Multiplying by One

For this, you need to understand two related things:

- If you multiply anything by one, it doesn't change.
- If you divide anything by itself (i.e., *a*/*a*), that's equal to one.

For example, let's convert 6 inches to centimeters. Start by writing down the thing you want to convert. To be super official, put it on opposite sides of an equals sign. like this:

$$6 \text{ inches} = 6 \text{ inches}$$

Now we'll multiply the right-hand side by 1. But we'll write the "1" in a special way. Suppose we know that 1 inch = 2.54 centimeters.

That means that $\frac{1 \text{ in}}{2.54 \text{ cm}}$ is equal to one, and so is $\frac{2.54 \text{ cm}}{1 \text{ in}}$.

We pick the one where the inch is in the denominator (the second one). Like this: $6 \text{ in} = 6 \text{ in} \times \frac{2.54 \text{ cm}}{1 \text{ in}}$

Now, if we have even a little bit of algebraic understanding, we can cancel the inches. Once we do that, we perform the calculation with the numbers (6 times 2.54, divided by one) and get our answer:

$$6 \text{ in} = 6 \cancel{\text{in}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{in}}} = 15.24 \text{ cm}$$

To recap, we multiply by one—which we can always do. But we write the "one" intentionally so that we can cancel the units that we are converting *from*—the units we're trying to get rid of (in this case, inches).

Let's do a more complicated one: let's convert 60 miles per hour to feet per second.

First we have to know that "miles per hour" is also $\frac{\text{mi}}{\text{hr}}$. Then finding the answer is a matter of knowing some conversions, multiplying by one repeatedly, and choosing just which "ones" we want.

$$\begin{aligned} 60 \text{ mph} &= 60 \frac{\text{mi}}{\text{hr}} \\ &= 60 \frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \times \frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} \times \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \end{aligned}$$

We began by using 1 mile = 5280 feet, and put the mile on the bottom so we could cancel the mile (red). Knowing that one hour is 60 minutes meant we could put the hour on the *top* to cancel it (green). But now our remaining units are "feet per minute," which is almost but not quite what we want. We need another conversion to change minutes to seconds:

$$\begin{aligned} 60 \text{ mph} &= 60 \frac{\text{mi}}{\text{hr}} \\ &= 60 \frac{\cancel{\text{mi}}}{\cancel{\text{hr}}} \times \frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} \times \frac{1 \cancel{\text{hr}}}{60 \cancel{\text{min}}} \times \frac{1 \cancel{\text{min}}}{60 \text{ sec}} \\ &= \frac{60 \times 5280}{60 \times 60} \frac{\text{ft}}{\text{sec}} \\ &= 88 \frac{\text{ft}}{\text{sec}} \end{aligned}$$

This technique is good for reinforcing students' understanding about canceling, and also for helping students see how the very idea of a unit acts just like an abstract algebraic symbol, and behaves correctly if you think of using a unit as *multiplication*. Fifteen centimeters is, in fact, 15 *times* a centimeter.

Substitution

Substitution takes a slightly different tack, but it might make better sense to some students. Let's convert 6 inches to centimeters again as our example. In this technique, you literally substitute 2.54 cm for the "in." Like this:

$$\begin{aligned} 6 \text{ in} &= 6 \text{ in} \\ &= 6\ (2.54 \text{ cm}) \\ &= 15.24 \text{ cm} \end{aligned}$$

See what we did? We replaced the inch at the top with something that's the same as an inch: 2.54 cm.

This seems simpler at first glance, but it can get harder. Suppose you're converting 10 centimeters to inches. You need to substitute for the centimeters. How many inches is a centimeter? You could look it up (0.394) or you might know that it has to be 1/2.54.

$$\begin{aligned} 10 \text{ cm} &= 10 \text{ cm} \\ &= 10\ (0.394 \text{ in}) \\ &= 3.94 \text{ cm} \end{aligned}$$

To do a "cascade" of conversions is easy, especially on a white board so you can erase. Repeating the lines for clarity, and putting in lots of parentheses, let's find out how many seconds there are in a day.

$$
\begin{aligned}
1\text{ day} &= 1\text{ day}\\
&= 1\,(24\text{ hr})\\
&= 1\,(24\,(60\text{ min}))\\
&= 1\,\big(24\,(60\,(60\text{ sec}))\big)\\
&= 1 \times 24 \times 60 \times 60\text{ sec}\\
&= 86400\text{ sec}
\end{aligned}
$$

Metric Conversions

We hope students see how great the metric system is, and how it makes conversion within the metric system easy. But some students still have trouble. Even if they see that “you just have to move the decimal point,” they can have trouble knowing how far and in what direction.

Here are two complementary strategies.

First, laboriously insist on either “multiplying by one” or “substitution.” And then ask if the student can see an easier way to do the conversion. Does the number look familiar? What does *kilo* mean?

Second, give students explicit estimation tasks to develop their sense of the units. For example, in a warm-up, ask them to estimate the width of the room in meters. (Or the height of a desk in centimeters, or the mass of the teacher in kilograms, or the mass of a nickel in grams…) Then, as a class, do a conversion, first asking, OK, you say the room is about 14 meters wide. Suppose we measured it in centimeters. Would there be *more* centimeters than 14, or *fewer*? Students will still jump to 14000 or 140 instead of 1400, but they will at least go in the right direction. When you do the careful calculation, help them see how they could have just “added two zeros” because they know that 100 centimeters make a meter.

Converting to kilometers (0.014 km) is harder, of course. In general, in class, convert to smaller units before converting to larger ones. Multiplying by one:

$$14\text{ m} = 14\text{ m} \times \frac{1\text{ km}}{1000\text{ m}} = 0.014\text{ km}$$