

Teacher Tune-up

Quick Content Refresher for Busy Professionals

How is a pendulum helpful in thinking about kinetic and potential energy?

Conservation of energy is a fundamental law of physics. Within a closed system, the total amount of energy remains the same—it just gets converted from one form to another. That’s the simple headline about energy. The more complicated news, of course, is that energy can take many forms, and most systems are not closed but open (i.e. energy enters or escapes). Add entropy to the discussion—the inevitable decay of useful energy into conserved but less useful forms—and things can get complicated fast. It would be nice if we could hold all this complexity at bay and see a simple, idealized illustration of energy being conserved as it changes form.

That’s where pendulums come in.

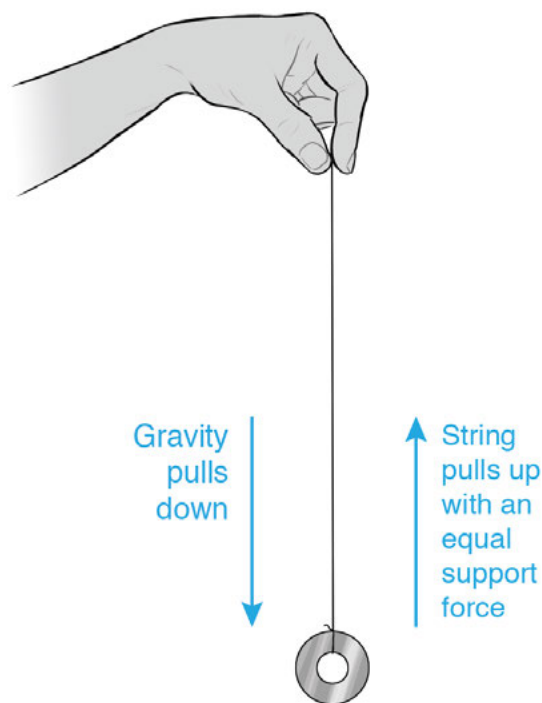
With a pendulum, the salient forms of energy (that is, the forms we’re interested in) are **kinetic energy** (the energy of motion) and gravitational **potential energy** (the potential an object has to do work by virtue of its position in a gravitational field). We’ll shorten “gravitational potential energy” to just “potential energy” here, although there are other forms of potential energy, like chemical potential energy. (We’re not interested in the potential chemical energy we could release by burning the string that supports the pendulum’s heavy bob!)

As a pendulum swings, its energy is continually converted back and forth between the kinetic energy of the moving pendulum bob and the potential energy that exists when the bob is positioned at the high point of its path. Ideally, the total energy is perfectly conserved.

(Of course, reality is not ideal: no real-world pendulum can be a perfectly closed system, and energy escapes gradually as heat because of air resistance and because of friction at the string’s fulcrum. If we were investigating entropy, this leakage of energy would be a salient point, and the slight heating of the environment as a pendulum gradually slows down would be part of a more complete accounting of the conservation of energy. Friction does not contradict the law of conservation of energy; rather, friction is an important part of how that law works in the real world.

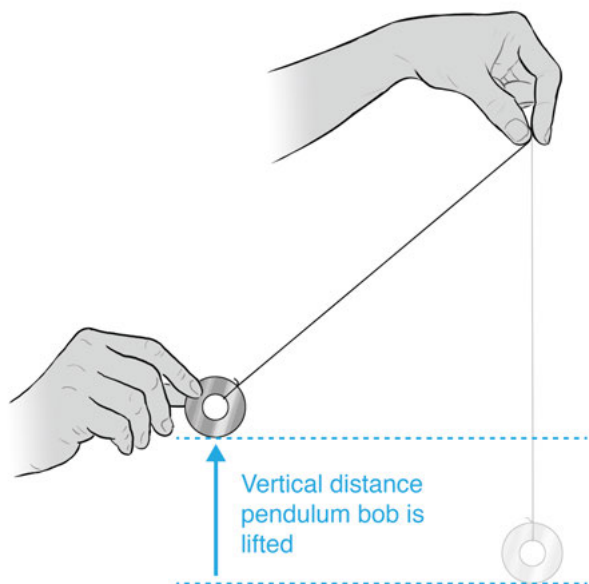
Nevertheless, real pendulums provide a clear enough approximation of *ideal* pendulums to let us contemplate energy conservation in terms of an idealized exchange between potential and kinetic energy.)

Let’s start with the pendulum hanging straight down. The force of gravity pulling the pendulum bob down is exactly balanced by the support force of the string pulling it straight up. With those forces canceling out, the net force on the pendulum is zero and it won’t start moving. Of course, if we let go of the string, removing the support force, the bob would fall, showing that it had potential energy in a larger frame of reference—a larger system. But within the system we’re interested in, with that support force from the string, the pendulum has no energy.

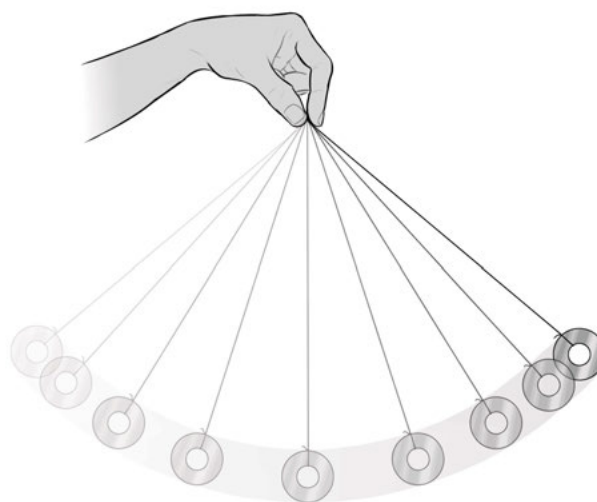


But what happens if we pull the pendulum bob up to one side and then release it? It will swing from side to side for a long time. Why?

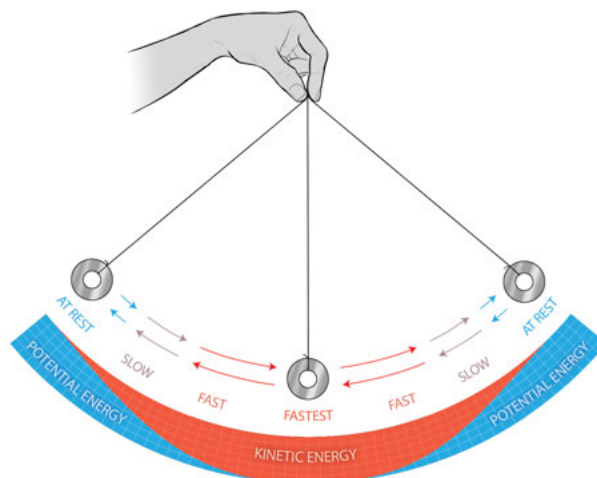
Pulling the bob up to one side requires some work against gravity, since the bob is now higher than its low point. That work gets added to the pendulum's system in the form of potential energy (note that the system is open initially: we're putting energy in before we "close" the system and watch it run). Since energy is the capacity to do work, and work (as the term is used in physics) equals force times distance ($W = fd$), the potential energy we invest in the system equals the weight of the bob (a force) times the vertical distance we lift it above its starting low point. (The horizontal displacement of the bob doesn't count here since moving it horizontally is not working against gravity.)



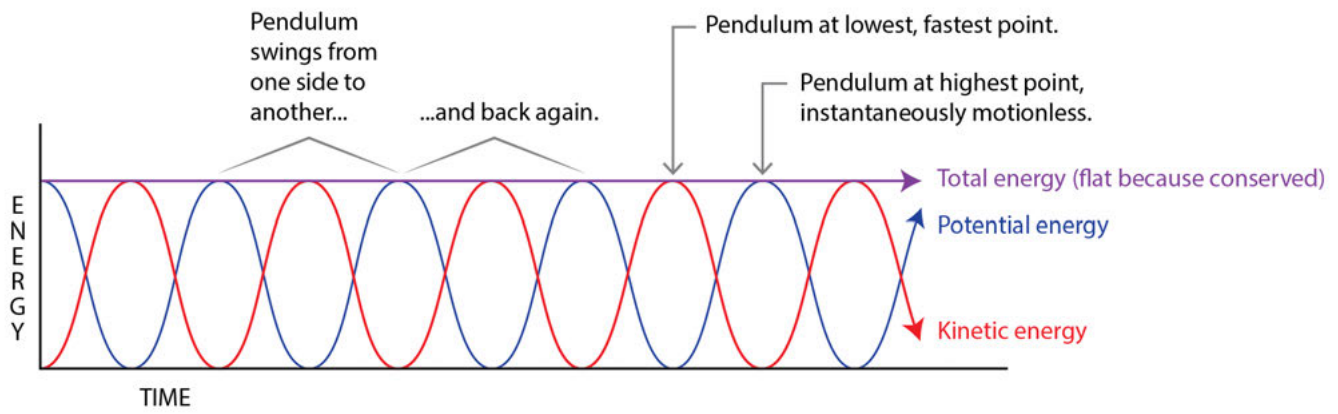
When you let go of the raised bob, its potential energy is converted to kinetic energy as it swings down with increasing speed, and then back to potential energy as it swings up on the other side with decreasing speed.



It then swings back the other way, and so on, its energy converting repeatedly back and forth between potential and kinetic. At the height of each swing, in the instant when the bob is motionless between upswing and downswing, the system has all potential energy and no kinetic energy. At the low point of its swing, when gravity has given the bob all the downward acceleration it can, all the energy is kinetic. The illustration below indicates how fast or slow the pendulum is moving at different points in its arc, and also what share of its total energy is potential and what share is kinetic at different points. The total energy is conserved throughout the swing, as suggested by the consistent width of the curved band that shows the changing ratio of PE and KE.

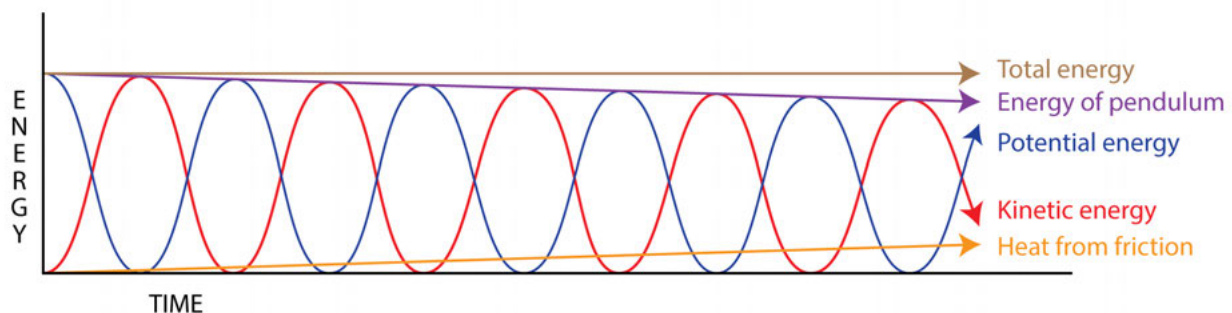
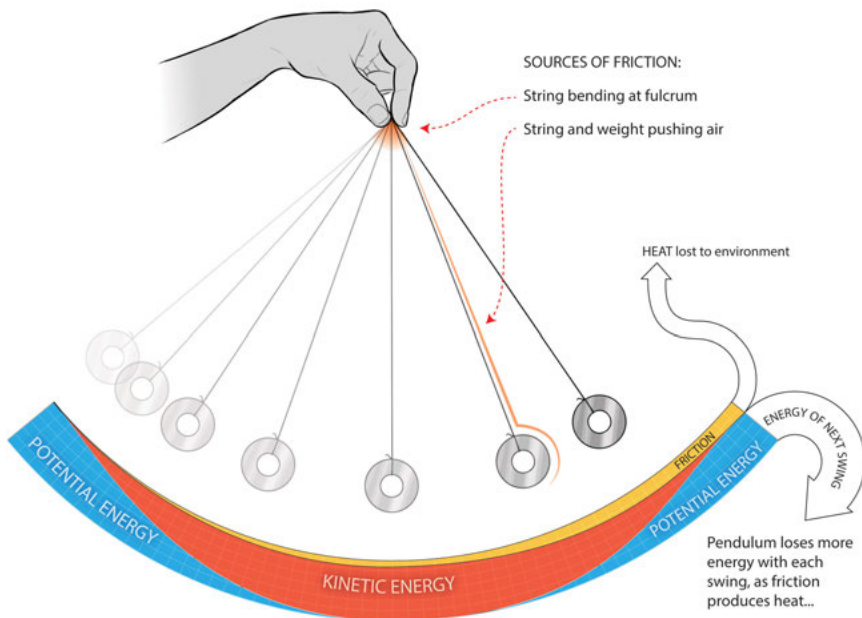


Below is a more conventional graph of the potential and kinetic energy in the system over time.

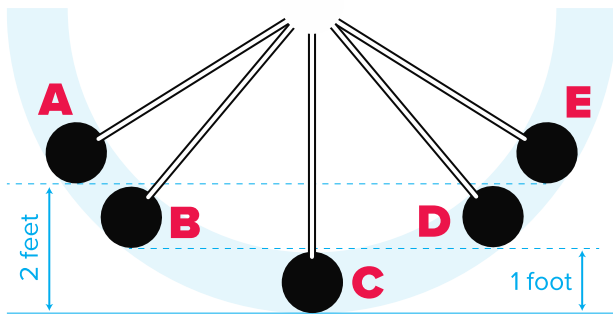


The purple line indicating *total* energy stays flat over time, because conservation of energy makes the exchange of potential energy (blue line) and kinetic energy (red line) a zero sum game. The blue curve (PE) is at its peak when the pendulum's bob is at its highest elevation, with an instantaneous velocity of zero; the red line (KE) is at its highest when the bob is at its lowest point, with maximum velocity.

As we've noted, real pendulums are not truly closed systems because they aren't immune to friction. The illustration and graph below revise their earlier counterparts with a tip of the hat to entropy.



Let's turn from images to specific numbers, and consider the following table. It shows quantitative information for a large pendulum with a 5-pound bob that's released from an elevation 2 feet above the lowest point of its arc. Speed in this table is measured in feet per second, and energy in foot-pounds (the amount of work it takes to lift a one-pound weight one foot).



Position	Height of bob (feet)	Speed of bob (feet per sec)	Potential energy (ft-lb)	Kinetic energy (ft-lb)	Total energy (ft-lb)
A	2	0	10	0	10
B	1	8.02	5	5	10
C	0	11.34	0	10	10
D	1	8.02	5	5	10
E	2	0	10	0	10

Pendulums are not the only illustration of conservation of energy. In a sense, *everything* is an illustration of conservation of energy. In the messy world beyond ideal pendulums, there are many more forms energy can take, and many of its transformations are hard to perceive clearly. But the simple pendulum provides a vivid reminder of what is going on in more complex ways all the time. The abstract and the concrete seem to meet in a pendulum. As we watch a pendulum's motion, our mind can swing back and forth between the concept of conservation of energy, and its perceptible embodiment. And for a hypnotic moment, we may feel that our conceptions and perceptions are forms of the same understanding.