Teacher Tune-up

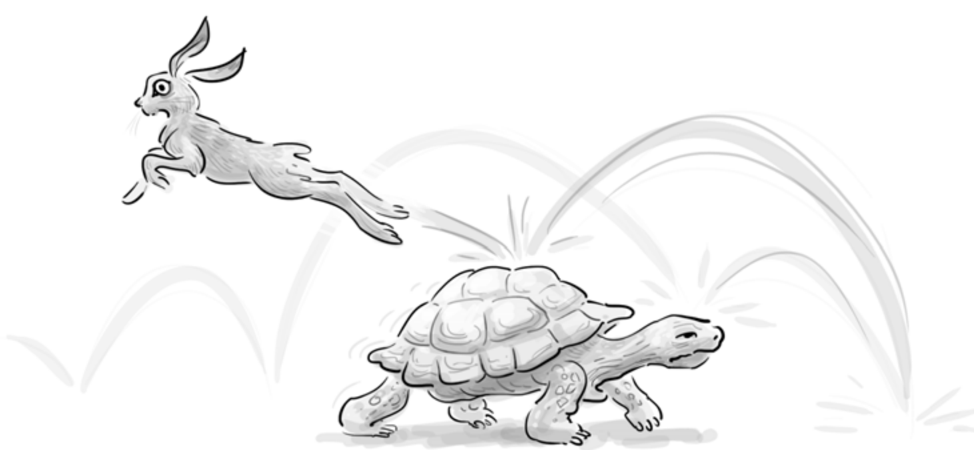
Quick Content Refresher for Busy Professionals

How do position, distance, time, speed, velocity, acceleration, force, mass, and momentum relate to one another?

Our everyday experience moving around, and pushing, pulling, dropping, or throwing objects, gives people a pretty good intuitive sense of the physics of motion. But sometimes the finer points of motion escape our intuitive grasp. The concept of acceleration, for example, didn't really exist before the seventeenth century. Physics steps in with a clear vocabulary and a few simple formulas that allow us to talk more precisely about motion.

Speed is a measure of *how fast* something moves. It's measured in terms of units of distance divided by units of time, or distance *per* unit time. Thus 88 feet per second, 60 miles per hour, and 331 meters per second (the speed of sound in air) are all *speeds*.

Velocity is the *speed and direction of motion* of an object. In other words, velocity is the rate of change of *position* of an object (where position incorporates direction by specifying whether the object is moving backwards or forwards on a line, or moving sideways or up and down in two or three dimensions). Quantities like velocity that include a direction as well as magnitude are *vector* quantities (as opposed to *scalar* quantities like speed, which only include magnitude).



In the illustration above, the tortoise moves steadily forward at a constant velocity (positive in relation to the race the animals are running); the hare is bounding about with changing velocity, sometimes positive, sometimes negative.

Expressed in equations, we could think of velocity as follows.

$$\text{velocity} = \frac{\text{distance and direction}}{\text{time elapsed}}$$

OR

$$\text{velocity} = \frac{\text{ending position} - \text{starting position}}{\text{end time} - \text{start time}}$$

Here's how a scientist calculates average velocity:

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$

In the formula above, v = velocity, x = distance from the origin, and t = time. The arrows over the v and x mean that they are vector quantities, so direction must be included.

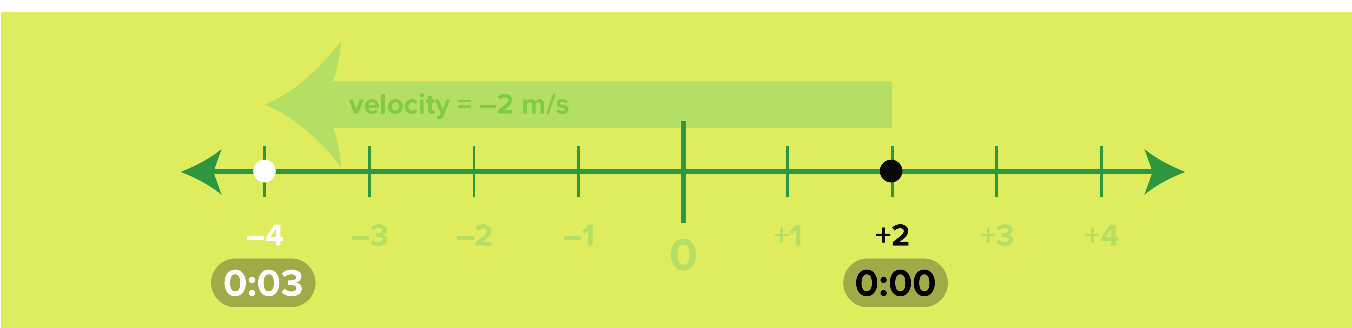
The delta symbol Δ means "change in," so Δt represents the change in time. You can also think of it as an instruction: take the final value of this quantity and subtract the initial value of this quantity. Like this:

$$\vec{v} = \frac{x_f - x_i}{t_f - t_i}$$

This formula defines average velocity. If a car starts from a standstill and speeds up until it is going 60 miles per hour, then slows down and comes to a standstill again, all the while going in the same direction, its actual instantaneous velocity (shown by the speedometer) will only be equal to its average velocity twice (once while the car speeds up and once as it slows down).

Usually, direction is defined by compass directions (North, East, Southwest, etc.), by degrees (–30°), or by using + and – signs to represent forward and backward motion. Two objects have equal velocities only when both speeds and both directions are the same.

Let's calculate velocity in one dimension:

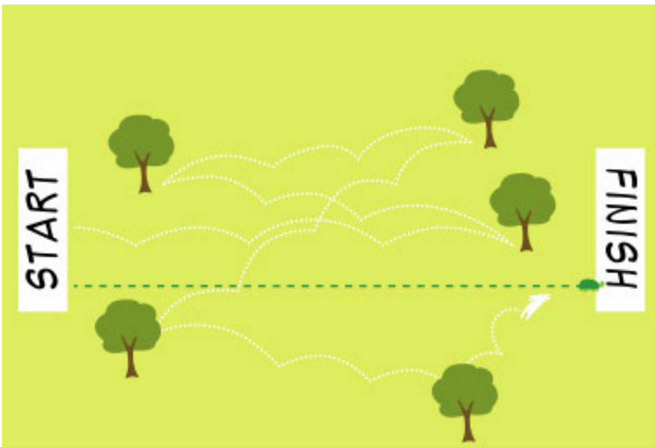


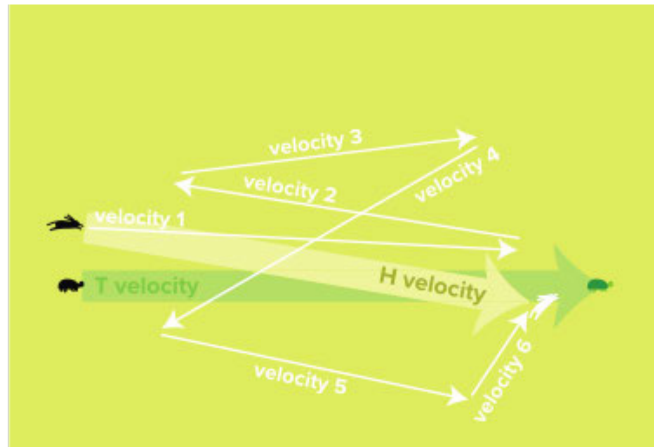
If an object moves from a position of +2 meters to -4 meters during a time of 3 seconds, its velocity is:

$$\vec{v} = \frac{(-4m) - (+2m)}{3s} = \frac{-6m}{3s} = -2\ m/s$$

In this case, the fact that the velocity is negative is important: it tells us the object is physically travelling backwards. The words “speed” and “velocity” are often used interchangeably in everyday conversation, but they are in fact not the same thing. Speed is *part* of velocity, but velocity is a vector quantity so it includes direction as well as speed. For example, when the speedometer on your car reads 30 mph it is only recording your speed because it does not tell you the direction of travel. If you know you are travelling 30 mph westward, then you know your velocity.

Let’s look again at the tortoise and a hare. Here’s what their paths might look like from above. The hare covers a lot of distance, but when you add the 6 velocity vectors together, the hare’s overall velocity falls short of the tortoise’s velocity. The tortoise wins the race! He went slowly, but never turned away from the finish line. The hare’s frequent negative and zero velocity, running back towards the starting block or resting under a tree, means that the hare’s average velocity does not measure up.





Both animals leave point A at the same time and arrive at point B at almost the same time. Our tortoise travels at a constant speed in a straight line from point A to point B, but the hare zigzags wildly around at various speeds. The tortoise and the hare make the trip with almost the same average velocity, and what’s more, the tortoise’s instantaneous velocity is the same as the average velocity at every moment in time. We know this because the tortoise is always traveling in the same direction: straight from the start to the finish.

How about the hare’s instantaneous velocity? Because the hare’s direction changes constantly, it’s possible that its instantaneous velocity was never the same as its average velocity! Maybe the hare was moving slower or faster than the average speed whenever it happened to be traveling in the same direction as the tortoise, who knows? The point is, average velocity doesn’t necessarily tell you anything about actual velocity at any given instant in time. (If we wanted to know our hare’s instantaneous velocity halfway through his crazy route, we might need to break out some calculus!)

All this change in velocity brings us to acceleration.

Acceleration is the *rate of change of velocity*, and is expressed in SI units of m/s^2. Acceleration can change an object’s speed, direction, or both. That expression, m/s^2, can be perplexing at first (or after going a long time without really thinking about what it means). It can be read “meters per second squared,” or “meters per second per second.” The equivalency of those two phrases is reflected in this equality:

$$\frac{m}{s^2} = \frac{\frac{m}{s}}{s}$$

The second way of writing the units may make it easier to see that it expresses a change in speed (or velocity when we include direction) over a period of one second. (But the standard m/s^2 version is easier to use in calculations.)

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time elapsed}}$$

OR

$$\text{acceleration} = \frac{\text{ending velocity} - \text{starting velocity}}{\text{end time} - \text{start time}}$$

Here's how a scientist calculates average acceleration:

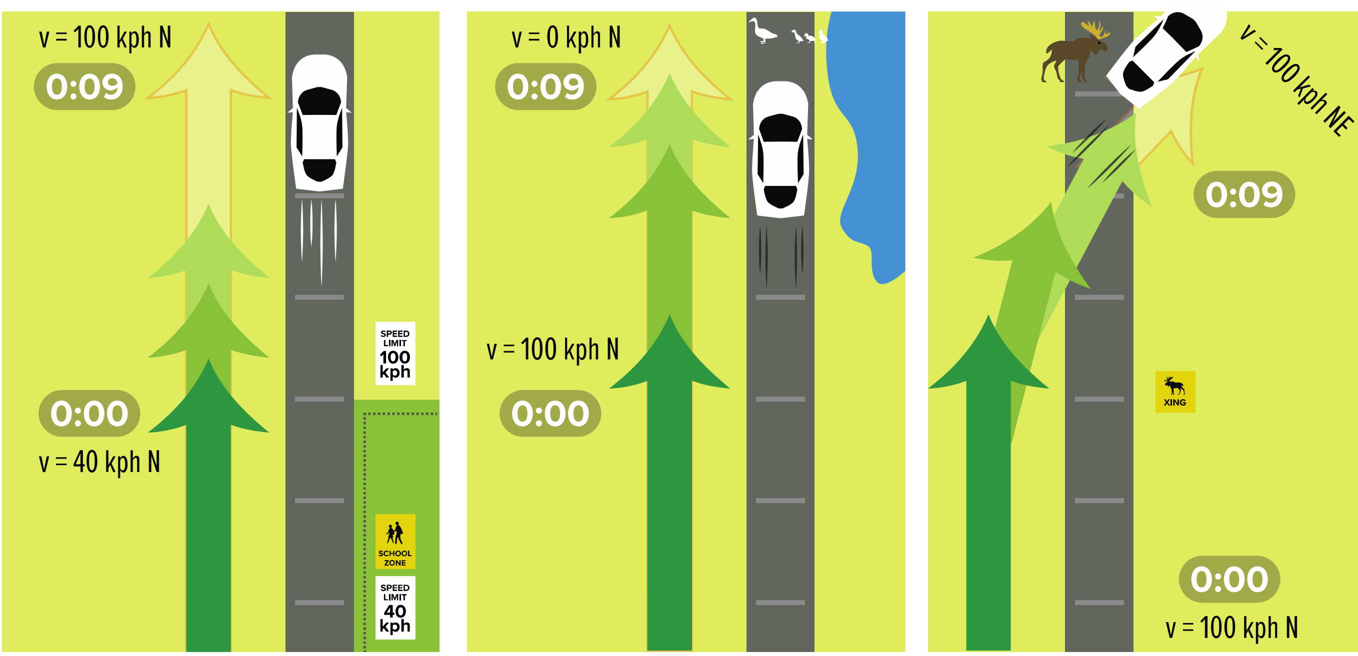
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

The word "deceleration" is often used for an object that is slowing down. Since acceleration uses direction, it is also a vector quantity.

This formula only tells us average acceleration. If a car starts moving from a standstill, and then 10 seconds later it is going 20 m/s (in a straight line), it has an average acceleration of $2 m/s^2$ (or 2 meters per second per second); but its actual acceleration could be erratic over those 10 seconds. As with our earlier wandering hare, we would need to use calculus to find the instantaneous acceleration of the car at 7 seconds after the start, if the driver were varying the pressure on the gas pedal. (Hypothetically, at 7 seconds the car could be going faster than 20 m/s and the driver could be using the brake to decelerate back down to 20 m/s by 10 second mark.)

There are three ways that an object can have an acceleration: by speeding up, by slowing down, or by changing the direction of its motion. The first two are caused by changes in the speed part of velocity, and the third by changes in the direction part of velocity.

To use driving a car as an example, you can have an acceleration by hitting the gas, hitting the brakes, or by turning the steering wheel. (Velocities below are given in kilometers per hour combined with a compass heading.)



Generally, a positive acceleration means the object is speeding up and a negative acceleration means the object is slowing down. Be careful though, because an object with a negative acceleration is not necessarily traveling backwards! For example, if a car is zipping down the highway at 60 mph (or 100 kph) and the driver sees a duck family crossing the road and applies the brakes, the car can be said to have negative acceleration (it is slowing down), but it is still going forward toward those adorable ducklings, at least for a while.

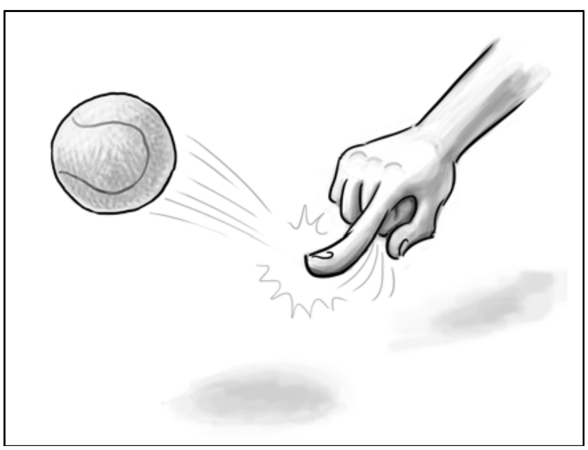
If an object is moving at a constant speed and in a straight line, like an airplane cruising due west at constant altitude and speed, then it has *zero* acceleration. Note that this doesn't mean forces aren't being applied to the airplane: the engines are providing forward thrust, and air resistance is applying force in the opposite direction; gravity is applying a downward force on the plane, and lift acts with an opposing upward force. As long as the forces cancel out to zero, the airplane does not accelerate and the velocity remains the same.

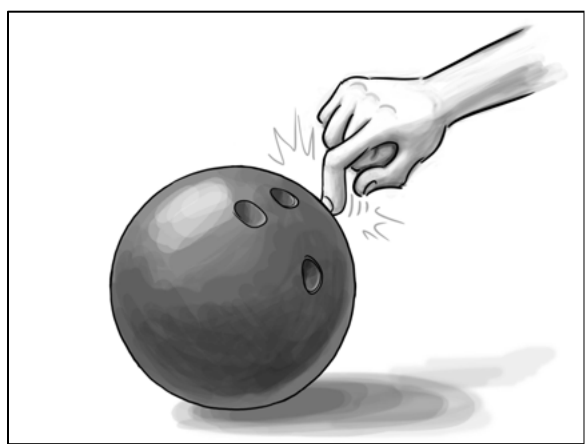
The velocity of an object cannot change unless a *net* force is applied. This notion may be counter-intuitive, because easy-to-overlook forces like friction and air resistance are usually slowing things down in our day-to-day experience. Because of friction, you have to keep pushing a car's gas pedal—somewhat confusingly named the "accelerator"—just to maintain a constant velocity on a level road. But in that situation, the car isn't really accelerating; the engine is just providing enough force to counteract the opposing force of friction. Friction has been fouling up people's thinking about motion since at least the time of Aristotle! A hockey puck gliding across ice with relatively little friction gives us a slightly more accurate image of the underlying laws of motion than does a car laboring along an asphalt road.

Newton's first law of motion corrects the false impression a world of invisible friction gives us. It states that ***an object continues in its state of rest, or of motion in a straight line, unless some force compels it to change its state of motion***. This law is also known as the law of inertia.

Inertia is a measure of *how resistant an object is to changes in its motion*. Inertia is measured by mass, using the SI unit of kilograms. Mass is a scalar quantity, meaning it does not include a direction.

A more massive object will resist changes in motion more, and thus have more inertia. For example, imagine flicking a table tennis ball with your finger, and then doing the same thing to a bowling ball. The low-mass table tennis ball has far less inertia, so it resists changes to its motion less and probably goes flying off as a result of the force from your finger. The high-mass bowling ball, on the other hand, barely budges because it has much more inertia and resists any changes to its motion.





The relationship between force, mass, and acceleration is summed up in **Newton's second law of motion.** This law states that ***acceleration is directly proportional to the net force acting on an object (and is in the same direction as the net force), and inversely proportional to the mass of the object.*** The law can be summarized as: acceleration = force/mass. Or just

$$a = \frac{F}{m}$$

Because it's customary to minimize fractions in physics laws, this is usually written as

$F = ma$

Momentum describes inertia in motion, and is calculated by multiplying an object's mass and velocity, using SI units of kg·m/s (kilogram-meters per second). Because it incorporates velocity, momentum is a vector quantity.

momentum = mass × velocity

Momentum is a *conserved* quantity. For example, if two objects collide, the total momentum of the objects (or their broken pieces) before and after the collision remains the same. After a head on collision between a fast moving truck (high momentum) and a slow moving car (low momentum), both vehicles may be moving in the direction the truck was initially going. However, the truck will have slowed down, and the overall momentum of both vehicles will add up to what it was before the collision. In a rear end collision, the vehicle in front will pick up some momentum, equal to the momentum the car in back loses.

When comparing the momentum of one object to another, it's important to realize that mass plays a role as well as velocity. If two objects have the same velocity, they do NOT have the same momentum unless they also have the exact same mass! This distinction becomes clear when you are hit in the shin by a table tennis ball, and then by a bowling ball, each traveling at the same velocity.